TWO BEAMS DUMB-BELL UNDULATOR ANALOG MODEL

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Abstract
Analogic simulation is more intuitive and also enables a validation of the numerical simulation. The undulator consists of a dumb-bell wires stack. In each wire the current circulates alternatively from a wire to another. The dumb-bell contour is implemented using an analog gate controlled by a gate signal. In free-electron laser (FEL) research and development one of the main trends is the elaboration of the compact devices. The undulator is the principal component where the phenomenon of coherent radiation take place. A new theoretical model of an undulator for free electron lasers in a dumb-bell magnetic field is presented. The undulator is a stack of wires which are described in xy plane by dumb-bell equations.

Keywords: free electron laser, electromagnetic undulator, analog simulation.

1. Introduction
In free-electron laser (FEL) research and development one of the main trends is the elaboration of the compact devices [1],[2],[3]. The undulator is the principal component where the phenomenon of coherent radiation take place.

A new theoretical model of an undulator for free electron lasers in a dumb-bell magnetic field is presented. The undulator is a stack of wires which are described in xy plane by dumb-bell equations (in polar coordinates), which is basically a pair of circles for the circular portion: 
\[ x = \rho \cdot \cos(\theta); \quad y = \rho \cdot \sin(\theta), \quad \theta = -\theta_0 \cdots + \theta_0 \] and for 
\[ \theta = \pi - \theta_0 \cdots \pi + \theta_0, \]
where
\[ \theta_0 = \frac{\arcsin\left(\frac{y_0}{\rho_0}\right)}{2} \]
is the limit angle that separates the circular and the linear portions of the curve. The linear portion of the dumb-bell curve has the next equations:
\[ x = -x_0 \cdots x_0; \quad y = y_0 \] for the upper region and \[ x = -x_0 \cdots x_0; \quad y = -y_0 \] for the lower region of the linear portion. Here \( x_0 \) and \( y_0 \) are the coordinates of the frontier point:
\[ x_0 = \rho \cdot \cos(\theta_0) \]
\[ y_0 = \rho \cdot \sin(\theta_0) \]
Fig. 1. Dumb-bell in cartesian coordinates (left) and polar coordinates (right)

The dumb-bell profile is used then to get the vector potential components \(B_x\), \(B_y\) and \(B_z\) by integrating over the wire profile. Dumb-bell wire stack is presented in figure 2.

Fig. 2. Dumb-bell wire stack

2. Method

The equations describing the model of the magnetic field generated by the undulator are the next. In the next equations \(x_p\), \(y_p\), \(z_p\) denotes the coordinates of the reference point, while \(z_c\) denotes the current position of the wire.

- for the linear portion of the domain:

\[
\begin{align*}
    c_1 &= (y_p - y_o)^2 + (z_p - z_c)^2 \\
    c_2 &= \frac{x_p - x_o}{c_1 \sqrt{(x_p - x_o)^2 + c_1}} \\
    c_3 &= \frac{x_p + x_o}{c_1 \sqrt{(x_p + x_o)^2 + c_1}} \\
    B_x &= 0 \\
    B_y &= (c_2 - c_1) \cdot (z_p - z_c) \\
    B_z &= -(c_2 - c_1) \cdot (y_p - y_o)
\end{align*}
\]

- for the circular portion of the domain:
\[
c_4 = R \cdot \cos(\theta) \cdot (x_p - R \cdot \cos(\theta))
\]
\[
c_5 = R \cdot \sin(\theta) \cdot (y_p - x_0 \cdot R \cdot \sin(\theta))
\]
\[
c_6 = -R \cdot \sin(\theta) \cdot (z_p - z_c)
\]
\[
c_7 = -R \cdot \cos(\theta) \cdot (z_p - z_c)
\]
\[
x_i = x_0 + R \cdot \sin(\theta)
\]
\[
y_i = R \cdot \cos(\theta)
\]

\[
B_x = \int_{\theta}^{\theta'} \frac{c_6 \cdot d\theta}{\sqrt{\left((x_p - x_i)^2 + (y_p - y_i)^2 + (z_p - z_c)^2\right)^{3/2}}}
\]
\[
B_y = \int_{\theta}^{\theta'} \frac{c_7 \cdot d\theta}{\sqrt{\left((x_p - x_i)^2 + (y_p - y_i)^2 + (z_p - z_c)^2\right)^{3/2}}}
\]
\[
B_z = \int_{\theta}^{\theta'} \frac{(c_4 + c_5) \cdot d\theta}{\sqrt{\left((x_p - x_i)^2 + (y_p - y_i)^2 + (z_p - z_c)^2\right)^{3/2}}}
\]

The limits of the integration are the frontier points between circular and linear portions. The two portions are implemented separately and then are mixed using an analog gate (74HC4053) controlled by a gate signal, shown in figure 3.

![Circle gate signal and its analog implementation](image)

Fig. 3. Circle gate signal (left) and its analog implementation (right)

Besides the numerical computation approach of the magnetic field generated by the undulator current, the other possibility is the analogical simulation. Such an approach is more intuitive and versatile and also enables a validation of the numerical simulation. For this reason the equations describing the model of the magnetic field generated by the undulator were, besides the numerical evaluation, simulated using an electrical analogical model. Analog computation was accomplished dividing the equations into small pieces that may be easily implemented using elementary functional analogic blocks. In fact the equation comprises mathematical operations like integration, multiplication, division, addition, substraction, square root, sine and cosine functions. All these may be simulated by an analogical circuitry, using appropriate functional modules. These functional modules are detailed in the next chapter.

### 3. Results

The model was built using some functional blocks, as: sine/cos generator, analog multipliers, analog dividers, square root device, operational amplifiers and integrators.
The sine/cos generator is a classical harmonic oscillator with both outputs (sine and cosine). Sine and cos of 2\( \theta \) and 3\( \theta \) were generated using some AD 633 analog multipliers and MC 1458 operational amplifiers. The square root was generated using a MC 1458 operational amplifier and an AD 633 analog multiplier. The division is achieved using a solution quite similar of the square root: the same MC 1458 operational amplifier and an AD 633 analog multiplier, but no diode at amplifier’s output. The integrator is also a classical one, using the same MC 1458 operational amplifier. Because of the symmetry, \( B_x \) and \( B_y \) equal to 0, so it remains only \( B_z \). The dependence of \( B_z \) of \( z \) is shown in figure 4.

![Figure 4](image)

Fig. 4. The undulator z magnetic field component vs. z direction.

### 4. Conclusions

In this paper a new model of an undulator for free electron lasers is presented. The current undulator structure is given by a series of dumb-bell wires. Each wire presents 90 degree symmetry. The magnetic field integrals components are analytical computed. The middle magnetic field aspect is mainly longitudinal. The transversal aspect was created by electrons with transversal components. This new treatment of the problem reduces the time and complexity for magnetic components evaluation for this structure. The model is thought for structures with two beams simultaneously. The analogical simulation for field magnetic components is easier and more quickly than the numerical one.

### References