ON THE DYNAMICS OF MOTIONS OF A ROBOT FOR THE ELECTRONIC INDUSTRY

Adrian CHIRIAC\textsuperscript{1}, Ramona NAGY\textsuperscript{1}, Liviu BERETEU\textsuperscript{1}, Alexandru BOLTOSI\textsuperscript{1}

\textsuperscript{1} Department of Mechanics and Vibrations, “Politehnica” University of Timișoara
Bd. Mihai Viteazu, nr.1, 300222 Timișoara, Romania

Abstract
In the paper, there are studied some aspects concerning the dynamics of motions of a robot, dedicated to the automatic implantation of electronic components in the printed circuit boards. The aim of study is to put in evidence the dynamic interactions between the motions of robot. It is also evaluated the influence of these interactions of the positioning precision of the manipulated electronic components.

Keywords: industrial robot, positioning precision, motion interaction.

1. Introduction
In the paper, there are studied some aspects concerning the dynamics of motions of a robot \cite{2}\cite{3}\cite{5}, dedicated to the automatic implantation of electronic components in the printed circuit boards. The aim of study is to put in evidence the dynamic interactions between the motions of robot. It is also evaluated the influence of these interactions of the positioning precision of the manipulated electronic components \cite{1}.

2. Dynamic Model
The dynamic model of the robot is presented in figure 1 \cite{4}\cite{6}: 1 – manipulated electronic component; 2 – prehension device; 3 - horizontal translation module; 4 – vertical translation module; 5 – rotation module, around the vertical axis.

3. Differential Equations of Motion
To establish the differential equations of motions of robot, the Lagrange equations of second species are applied, \[ \frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}_i} \right) - \frac{\partial E}{\partial q_i} = Q_i, i = 1,2,3, \] where, as generalized coordinates, there are chosen: \( q_1 = x \) - abscissa of center of gravity \( C_3 \) of the arm of horizontal translation, \( q_2 = z \) - appicate of center of gravity \( C_4 \) of the module of vertical translation, \( q_3 = \theta \) - angle
of rotation of the rotation module, around the vertical axis $Oz$. Also, in the equations, $E$ is the total kinetic energy of robot, $E = \frac{m_1 + m_2 + m_3}{2} (\dot{x}^2 + \dot{z}^2) + \frac{m_4}{2} \dot{\theta}^2 + \frac{1}{2} [J_{1c} + J_{2c} + J_{3c} + J_{4z} + m_1 (x + \frac{l_3}{2} + l_1)^2 + m_2 (x + \frac{l_3}{2})^2 + m_3 x^2] \dot{\theta}^2$, where: $m_1$ - mass of manipulated electronic component; $J_{1c}$ - axial moment of inertia of manipulated electronic component; $l_1$ - distance between the gravity centers $C_1$ and $C_2$ of the manipulated electronic component and the prehension device; $l_3$ - length of the arm of horizontal translation; $m_2$ and $J_{2c}$ - mass and moment of inertia in relation to the vertical axis passing by $C_2$ of prehension device; $m_3$ and $J_{3c}$ - mass and moment of inertia in relation to vertical axis passing by $C_3$ of horizontal arm; $m_4$ and $J_{4z}$ - mass and moment of inertia in relation to the $Oz$ axis of vertical translation module; $J_{4z}$ - moment of inertia in relation to the $Oz$ axis of rotation module.

For the generalized forces $Q_i, i = 1, 2, 3$, there are obtained the expressions $Q_1 = F_{Mx}$, $Q_2 = F_{Mz} - G$, $Q_3 = M_M$, where: $F_{Mx}$ - motive force for the horizontal translation; $F_{Mz}$ - motive force for the vertical translation; $G$ - total weight of manipulated electronic component and translation modules; $M_M$ - moment of the motive torque for the rotation.

Figure 1: Dynamic model
Taking into account these expressions and introducing the notations

\[ m_x = m_1 + m_2 + m_3, \quad m_z = m_1 + m_2 + m_3 + m_4, \quad l_z = \frac{m_1(l_1 + 2l_x) + m_2l_z}{2m_z}, \]

\[ J_z = \frac{J_{sc} + J_{2c} + J_{sc} + J_{4c} + J_{5c} + m_1\left(\frac{l_3}{2} + l_z\right)^2 + m_2\left(\frac{l_3}{2}\right)^2}{2}, \]

the equations (1) become

\[ m_x \ddot{x} - m_z \dot{x}^2 \ddot{\theta} = F_{Mx}, \quad m_z \ddot{z} = F_{Mc} - G, \quad J_z \ddot{\theta} + 2m_z \dot{x} \ddot{\theta} + m_z \dot{x} \dot{\theta} + m_z \dot{x} \dot{\theta} = M_M. \]

By analyzing the equations, it results that between the motion of the vertical translation module and the motions of the other two modules, there are not dynamic interactions, because its equation of motion (the second one in), is decoupled of the other ones. On the other hand, between the motions of the horizontal and rotation modules, there are important dynamic interactions. In order to put in evidence the influence of these interactions on the positioning precision of the manipulated electronic component, there are considered small perturbations \( \Delta x, \Delta z, \Delta \theta \), around of its position, \( x = x_0 + \Delta x, \quad z = z_0 + \Delta z, \quad \theta = \theta_0 + \Delta \theta \), where \( x_0, z_0, \theta_0 \) are the cylindrical coordinates of the considered position in the considered working space of the robot. By introducing these expressions in the equations and neglecting the small infinities of second degree, it results the system of differential equations in perturbations, \( m_x \Delta \ddot{x} = \Delta F_{Mx}, \quad m_z \Delta \ddot{z} = \Delta F_{Mc}, \quad [J_z + m_z(x_0 + l_z)] \Delta \ddot{\theta} = \Delta M_M \).

4. Analysis of Steady-State Regime of Motion of Robot

To analyze the steady-state motions of the robot, which are made with constant velocities \( v_{x0}, \quad v_{z0}, \quad \omega_0 \), there are considered their small perturbations \( \Delta \dot{x}, \quad \Delta \dot{z}, \quad \Delta \dot{\theta} \):

\[ \dot{x} = v_{x0} + \Delta \dot{x}, \quad \dot{z} = v_{z0} + \Delta \dot{z}, \quad \dot{\theta} = \omega_0 + \Delta \dot{\theta}. \]

By introducing these expressions in the equations and neglecting the small infinities of second degree, it results the system of differential equations in perturbations: \( m_x \Delta \ddot{x} - 2m_x \omega_0 (v_{x0} \dot{t} + x^* + \Delta x) \Delta \ddot{\theta} - m_x \omega_0^2 \Delta x = \Delta F_{Mx}, \quad m_z \Delta \ddot{z} = \Delta F_{Mc}, \quad [J_z + m_z(v_{x0} \dot{t} + x^* + l_z)] \Delta \ddot{\theta} + 2m_z \omega_0 (v_{x0} \dot{t} + x^* + \frac{l_z}{2}) \Delta \theta + 2m_z \omega_0 (v_{x0} \dot{t} + x^* + \frac{l_z}{2}) \Delta \dot{x} + + 2m_z v_{x0} \omega_0 \Delta x = \Delta M_M, \) where \( x^* \) is the abscissa of the initial position of the manipulated electronic component. The obtained equations are linear, with variable coefficients. These ones can be considered approximately constant if, on the duration \( t' \) of the transitory process,
they are slowly variable. Thus, for the coefficient \( v_{\alpha \omega} t + x^* + \frac{l}{2} \), it results the condition
\[
v_{\alpha \omega} t^* \leq \lambda (x^* + \frac{l}{2}),
\]
where \( \lambda = 0.1 - 0.2 \), and for the coefficient \( J_s + m_s (v_{\alpha \omega} t + x^* + l_s) \), it is obtained the condition
\[
v_{\alpha \omega} t^* \leq -(x^* + \frac{l}{2}) + \sqrt{(x^* + \frac{l}{2})^2 + \lambda[\frac{J_s}{m_s} + x^* (x^* + l_s)]}.
\]
In all expressions above, the signs of the velocities \( v_{\alpha \omega} \) and \( \omega_0 \) determine the direction of the resultant moment of the complementary Coriolis forces of inertia.

5. Conclusions
By analyzing the differential equations in perturbations, it remarks that they are decoupled. The physical consequence is the fact that the positioning precision of the manipulated electronic component can be separately calculated, in relation to each coordinate and that it is not influenced by the interactions between the motions of the modules of robot.

References